1. In Chapters I and II you studied a family of functions with the form $y = kx^n$. Members of that family include the straight line (for which n = 1), the parabola (n = 2), the hyperbola (n = -1), the square-root curve (n = 1/2), and many others. Then in Chapters III and VII you encountered the "trigonometric" functions, which includes sines, cosines, and tangents. *Sketch some examples.*

2. Imagine opening a savings account in a bank by making a single deposit. Although you never put in or withdraw any money after the initial deposit, the bank will pay "interest" money into the account. The amount of interest paid during a given time interval is proportional to the amount of money already in the account. We shall assume that the interest rate remains constant. Let "i" represent the interest rate, in decimal form. (For example, if the interest rate is 5%, then i = 0.05.)
   a. Sketch a graph of the amount in the account vs. time, and extend it back into the second quadrant.
   b. At the point where it crosses the vertical axis its value must be _______ than 0 and the slope of its tangent line must be ______ than 0. *Please illustrate by drawing that tangent line.*
   c. Is this graph a member of either of the families described in #1? ___
   If so, please write an equation to describe it and show that your equation is correct.
   d. How can you use the interest rate ("I") to predict the ratio of the amount in the account one year from now to the present amount? (Please *simplify* and define your symbols.) Ratio = _______
   e. Let's call that ratio the annual "growth factor", or "GF". Does the GF have units? ___ -Does the GF depend on the amount in the account? ___ -Does the GF always keep the same value? _____
   * f. Repeat 2a - 2e for a *negative* interest rate.

* 3. Which graphs from the capacitor experiment resemble the graph you made for #2f? -Is it just a superficial resemblance, or are they both the same type of curve? One way to find out is to see if the graph from the experiment has properties similar to those of the negative-interest graph. Use your data to find out if one of those graphs has the property described in 2e, and describe that property in your own words. Use 90 seconds instead of a year. Explain your conclusion clearly and quantitatively. *Uncertainty estimates or ranges are necessary in such an explanation.*

4. The relations described in #2 & #3 are examples of "exponential functions". A function is said to be "exponential" if it has a constant growth factor or decay factor or interest rate. Does 2e agree? ___ Name some other processes of growth or decay having that same property.

5. The "fractional change" in a quantity is its change in value divided by the value it had just before the change. For example, if a price increases from $20 to $22 then the fractional change in price is 2/20, or 0.10. If you multiply a fractional change by 100 you get a "percentage change".
   a. Sketch a graph to describe some exponential process of growth or decay. *Give it a title.* Mark a short interval on the time axis near its beginning and an equal time interval later on in the process.
   b. Define a symbol to represent that short time interval, and symbols *with subscripts* to represent the heights of the points marked on your curve. Use them to label the four values on the vertical axis.
   c. How must the fractional change during the first interval compare with the fractional change during the second interval? _______ -Does 2e agree?___
   d. Write your answer to 5c as an equation. (Use the symbols chosen above.) _____ = _____
   e. Divide each side of that equation by the time interval: ______ = _____
   f. You now have an equation which relates the *values* of the function at two points to the *slopes* at those two points. (If the interval is short, we needn't worry about the distinction between the slope of a tangent line and the slope of a chord.) Using the standard symbol for "rate" that was introduced on page 54b, (reviewed in #6 on page 100) rewrite the equation to show how the ratio of the two slopes is related to the ratio of the two values. *Don't ignore the title.* (____/____) = (____/____)
   * g. Conclusion: In any exponential process the rate of growth or decay depends on the present value of the growing quantity in a simple way. Write a sentence describing that relation. (Use #5 on RSI.) *Try not to confuse the cause with the effect.*
   h. Is 5g the same as the statement which you tried to paraphrase in #3? _____
   i. Is it correct to say that *any* growth or decay process satisfying 5g is exponential? _____
   j. Could your answer to 5g be called a definition? ____ -What word or phrase does it define? _______
   k. A copy of that improved definition is being saved for future use on RS __. *Does 5j contradict 5i?*

1. Make a data table and a graph of the equation $y = 2^x$. (Use x = 0, 1, 2, 3, -1, -2, etc.)
2. Is this an "exponential" function, according to the definitions on page 102 and 103? ____
3. Do the same for $y = 10^x$. Is this function also exponential? ____
4. Do all functions with that form have one point in common?___ --What point is that? (__ , __)
5. Sketch the derivatives of the functions in #1 & 3 and estimate the values of their y-intercepts. (Write those estimates onto the graphs, with explanations.)
6. Let "b" represent a constant: Is it correct to say that the function b^x is exponential for *ANY* positive value of "b"? ___ If not, please list the exceptions.
7. The number represented by "b" in #6 is called the "base" of the exponential function.
   a. When the base was 2, the y-intercept was ___ and the y-intercept of the derivative was approximately ___, which is _____ (higher, lower) than the original function's y-intercept.
   b. When the base was 10, the y-intercept was ___ and the y-intercept of the derivative was roughly ___, which is ____er than the original function's y-intercept.
   c. Is there a special "b" value for which the derivative has the *same* y-intercept as the original function? ___ If so, that special base will be called "e".
   d. According to 7a and 7b, the value of "e" must be between ___ and ___. You can make this range narrower by playing with your calculator. If you do, please explain clearly.

* 8. Use what you have already learned to prove that an exponential function with that special base called "e" is **identical to its own derivative** for all values of x. Please ask for help if necessary.

9. We have seen that the resistor voltage vs. time graphs in the capacitor experiment strongly resemble exponential functions. Let's start with the simplest exponential function of all, (y = ___, from #8) and see what modifications are necessary to make it describe one of those graphs accurately:
   a. We must reverse the x-axis to make our equation describe an exponential *decay* instead of a growth. That can be accomplished by reversing the sign of the "__" in our equation, so it says y = _____.
   b. We are discussing a function of *time*, not of "x", so we must replace the "x" with a "kt". (The adjustable constant "k" enables us to stretch or compress the graph horizontally by any desired amount. It also can make the exponent dimensionless.) Now we have y = _____.
   c. We use the letter "V" to represent voltage, so let's replace the "__" with a "V": __ = _____.
   d. The initial value of the voltage is not necessarily equal to 1.00; it could be any number of volts. We must insert a factor "V_o" which stretches the graph vertically by the desired amount and also makes the equation dimensionally consistent: **V** = ____.

   * e. Use the definitions of current, resistance, and capacitance to create similar equations describing the graphs of current, charge, and capacitor voltage vs time for the charging and discharging processes. Illustrate each equation with a sketched graph.
10. Devise a convincing test to see if the equations developed in #9 are correct. Perhaps you will be able to find the "k" value at the same time. (It may help to find out what "natural logarithms" are.)
11. In #9 you showed that ANY exponential function can be described by the equation $y = Ae^{kx}$, where "A" and "k" are constants chosen to fit the data and "e" has the special value described in 7d. Now you are going to create a formula for the **derivative** of such a function.
    a. If "A" and "k" are both equal to 1, then the derivative is y' = ____, as in #8.
    b. If "A" is not equal to 1, we use #2 on RS VIII, which says ___ = _______. We obtain y' = ____.
    c. If "k" is also not equal to 1, we use #3 on RS VII, which says ______. We obtain y' = _____.
12. Review the definition of "integration", which was introduced in Chapter II. Figure out how to integrate the general exponential function described in #11. Explain your process clearly. Remember to check by using #11. Remember to ask for help if you need it.

1. Imagine a one-farad capacitor charged up to one volt, with a resistor connected between its terminals. Nothing else is connected. Sketch the circuit, put polarity signs on the capacitor plates, and label it as a “charging” or a “discharging” circuit as on page 98.
   a. Electrons in a resistor always flow from regions with ____ pressure to regions with _____ pressure. (high, low) Indicate that flow direction in the resistor in your diagram.
   b. What kind of curve do you expect the voltage vs. time graph to be? (Sketch and name)
   c. The initial slope of the voltage vs. time graph is -1.00 V/s. Use the second capacitor law to calculate the initial current and the resistance of the resistor. (See #2 on RS XI.)
   d. Use that slope to figure out how much time it will take for the voltage to decrease by one percent, i.e. to 99% of its initial value. Also *show* (with a formula) how that amount of time was calculated from the given initial voltage and the initial slope given in 1c.

2. At any time during the discharging process in #1 you can figure out what the new voltage will be 0.01 seconds later by multiplying the present voltage by _____. (That factor decreases it by one percent.)
   a. To find the new voltage after 0.03 seconds, you could multiply the present voltage ___ times in a row by that factor mentioned in above, or by that factor to the ___ power.
   b. To reduce the voltage to *one half* of its original value you must multiply the original voltage by that factor ____ times in a row, or by the factor to the ____ power. (Use a calculator to find out.)

3. The "Half-Life" of an exponential process is the time required for it to go halfway to completion.
   a. According to 1d and 2b, the half-life of *this* capacitor's voltage-time graph must be _____ ____
   * b. How did you use the given initial value and initial slope to estimate the half-life in 1d?
   c. If we modify the circuit in #1 so that the voltage decreases by ***2%*** (instead of 1%) during each 0.01-second interval then the half-life will ___crease to _____ times its original value.
   d. A different modification causes the voltage to decrease by only *0.5%* during each 0.01-sec. interval. As a result, the half-life must ___crease by a factor of ___ . *Sketch graphs to illustrate 3c & 3d.*
   e. Measure the half-life of the exponential graph that you saved from page 99. Write the result *on* that graph. Also record the measured values of the capacitance and resistance that you used. *ALL of those results must be expressed in RANGE FORM, with SI units.* Staple the graph to this paper.

4. Suppose we double the resistance in circuit #1 without changing the initial voltage or capacitance: Ohm’s equation tells us that the initial current will then be ___ times what it was in #1, so the amount of time needed for a 1% change in Q or V_o will be _________ed, as in # ___ above. (3c or 3d?)

5. Suppose we double the capacitance without changing the initial voltage or resistance : How will that action affect the amount of time needed for a 1% change in Q or V_o? Explain briefly.

6. Suppose we double the original voltage *without* changing the resistance or capacitance:
   a. Ohm’s equation tells us that the initial current will then be multiplied by ___.
   b. The amount of charge transferred during the first 0.01 sec. will be ______ed.
   c. How will the new *percent* change in voltage compare with the original one? ____ *Use 17f on RS II.*
   d. The new half-life must be ___ times the original one. -Do 6b, c, & d contradict each other? ____

7. According to #4 & 5, half-life must be proportional to the _______ of R and C. (sum, product, quotient) Will there be any *other* variables in the half-life formula? ___ -Does #7 contradict 6d? ___

8. Using "k" to represent the proportionality constant, write the complete half-life formula that you deduced above: H = ______ -Does #8 contradict #7? ___ This discovery is saved in #__ on RS __.

9. Using #1, 7, and 3a above and simplifying units, we find that "k" is approximately _____. How many significant digits does this estimate have? ___ (Saved in # ___ on RS ____)

* 10. Use #8 & 9 with the R and C values that you recorded on the graph in 3e to predict your circuit's half-life, in *range* form, with SI units. *Remember to show how. Round off properly.* Then explain how the measured half-life in 3e supports or refutes that prediction.

11. A capacitor discharging through a 10 K resistor goes from 8.8 V to 2.2 V in 2.8×10^{-4} sec.
   a. How much time will this capacitor need to go from 2.2 to 1.1 volt with the same resistor? ______ ___
   b. Use 8-10 with 11a to determine its capacitance in microfarads. ________ µfd. *Show how.*
   c. You will need this new trick when you do page 108. Where are you keeping a copy? ________

1. You have already seen that when a variable is decreasing exponentially it does not change at a constant rate. If we suddenly forced its rate of change to become constant, the variable would continue decreasing steadily from that time, no longer following an exponential pattern. The time required to go from that point to zero in this linear fashion is called the "relaxation time" of the process.
 * a. Use that definition to prove that the relaxation time is equal to the ratio of the value of the variable at any time to its rate of change at that same time.
 * b. Measure the relaxation time on one of your exponential decay graphs as precisely as you can. Do it several times, starting at several different points. Also estimate the uncertainties of these measurements and explain your estimates.
 * c. Do you get significantly different results for different starting points, or does relaxation time seem to be a constant for a particular exponential decay?

2. Consider a capacitor discharging through a resistor. From page 102b, $V = V_o e^{-kt}$.
   a. Please define each symbol in that equation.
   b. To determine the rate at which the voltage is changing, we must ___________ each side of the equation. (This operation was explained on 102b. If you remember how to do it, skip to 2j.)
   c. That means we must find the limit of dV/dt as ___ approaches ____. (Here the "d" represents a "delta".) If we replace the "V" with the formula given above, we get "the limit of d(____)/dt."
   d. We can factor out the constant to get ___ times the limit of d( )/dt.
   e. Now replace the "kt" with an "x". We get V = ___ times the limit of d(____)/dt.
   f. The definition of "e" on p. 102b tells us that $d(e^x)/dx$ is equal to ____.
   g. But we did not have a "dx" in the denominator of part e; we had a "___".
   We can solve that problem by multiplying the right side of our equation by "dx/dx".
   That result can be written as the quotient of two products: **dV/dt = (_______)÷(_______)**
   h. If you reverse the order of one of those products you get an expression that is much easier to handle.
   Write it as the product of two quotients: dV/dt = (/   )(/   )
   i. Now one factor on the right side represents the slope of the linear graph of ___ vs ___.
   We know it is equal to ___, by definition. The other factor equals ____, as we mentioned in part f.
   j. We conclude that dV/dt = ______, so the relaxation time must be _____ divided by ____.
   (Use expressions involving k, e, t, and V_o ONLY.)
   k. After routine simplifying, our formula relating relaxation time to "k" says T_R = ______.

3. The "time constant" of a series R-C circuit is defined as the product of R and C.
   a. Prove that the units of this product are equivalent to time units.
   b. Calculate the time constant of your circuit. Remember to estimate its uncertainty.
   c. Compare the time constant with the relaxation time and make a comment.

4. It's easy to discover a relation between a circuit's time constant and its relaxation time:
   Consider a graph of charge vs. time for a capacitor discharging through a resistor.
   a. How is the resistor current related to the resistor voltage? I = _______
   b. How is the capacitor's discharge rate related to the current? dQ/dt = ________
   c. Combine the first two equations to relate dQ/dt to V and R: dQ/dt = ________
   d. How is the resistor voltage related to the capacitor voltage? Vr = ________
   e. Use the capacitor law to eliminate V from 4c. dQ/dt = ________
   f. Beginning at any point (Q_1, t_1) on the charge-time graph, draw a tangent line.
   Extend it to intersect the time axis. Call that point of intersection "t_2".
   What name have we already assigned to the time interval between t_1 and t_2? ____ ____
   g. How can the slope of the tangent line be expressed in terms of Q and that interval? ____
   h. How is that slope related to dQ/dt? ____
   i. Use parts "e", "g", and "h" to discover the exact relation between relaxation time and time constant.
   Write it here and also on your review sheet: _________ = __________

5. Determining the proportionality constant in the relation between half-life and time constant:
   a. Copy the equation of your resistor voltage vs. time graph from #2: V(t) = _____
   b. Let "H" represent the half-life. Part "a" says that when t = H, the resistor voltage will be: ___.
   c. The definition of "half-life" says that expression 5b must equal the original voltage times ___.
   d. Write 5c as a single equation relating H and k: ___ = ____ Solve it for H: H = _____ __
   e. Use 2k to replace the "k" in 5d with an expression involving other constants: H = _______