SKILLS YOU WILL NEED FOR THE SECOND APRIL TEST (1999, L-H)

1. Use algebra to solve a simple equation for its unknown. Plug in given data to determine the value and units of that unknown quantity, rounding off properly, as on every previous test.

2. Solve simple proportion problems by using #5 on RS I.

3. Calculate with scientific notation and units. Make uncertainty estimates in absolute form, in range form, and in percentage form.

4. Use #26 on RS II to estimate the uncertainty of a sum, difference, product, quotient, or square root.

5. Given the radius, diameter, circumference, or area of a circle, calculate any of the remaining three quantities. (See RS II.)

6. Use the definition of "gravitational field strength" on RS III. Use the well-known value for the earth's gravitational field strength which you recorded on RS III.

7. Use the energy conservation law and the energy formulas on RS VI.

8. Given the radius of a circular path and the amount of time needed to travel around it once, calculate the average speed and frequency. (Use 5 and 12 on RS VII.)

9. Use the centripetal acceleration and force formulas in #7 & 8 on RS VII.

10. Given any graph with a familiar shape, sketch a graph of its derivative or integral, as in Ch. II.

11. Use the definitions of “exponential process” on RS XI. Given any process in which the rate at which a variable changes is proportional to the value of that variable, sketch a graph describing how the variable changes as time goes on.

12. Given the initial rate and value of an exponential process, calculate the half-life. (#13 on RS XI)

13. Use the relation between water mass, temperature change, and heat transferred as you did on pages 88 & 89. Also use the definition of "change in temperature" as recorded on RS I.

14. Use the definition of “capacitance” on RS XI.

15. Use the capacitor laws as in #5 & 6 on RS XI:
    a. Given a capacitor's voltage vs time graph, describe its current vs time graph.
    b. Given a capacitor's current vs time graph, describe its voltage vs time graph.

16. Use the definition of electric current to calculate the amount of charge passed through a wire during a given time interval as we did on pages 99, 99R, and the previous test. (#3 on RS XI.)

17. Given the amount of charge that has entered or left a capacitor plate and the initial and final capacitor voltages, calculate the capacitor’s change in stored energy as in #7 & 8 on page 101.

18. Use the capacitor energy storage formulas in #7 on RS XI, as on the previous test.

19. Sketch graphs of voltage vs. current to describe the behavior of a resistor, a light bulb, or a battery. (Copy from RS X.)

20. Use Ohm’s equation and the definitions of "static resistance" and "dynamic resistance" on RS X.

21. Sketch a sine curve or a cosine curve. Sketch its derivative, as on page 101. (See #4 on RS VIII.)

22. Given the amplitude and frequency (or period) of a sinusoidal function, calculate the amplitude of its derivative. (Use 9 on RS VIII.)

23. Use #6 & 14 on RS XI to predict how a capacitor’s current amplitude is affected by a given change in frequency.

SECOND APRIL TEST (1999, LEVEL H) name:_____________

1. An auto manufacturer wants to make a television commercial in which a new car is driven around a vertical loop on a special track so that it is upside-down at the highest point. At that moment the normal force exerted on the car by the road will be downward. The strength of that normal force clearly cannot be less than zero, so the car's centripetal acceleration at the top must be greater than "g". The car's top speed is only 35 m/s on the level roadway approaching the loop. The stunt driver wants to be able to coast all the way around the loop because the engine sometimes stalls when going uphill.
   a. Let "S_b" represent the speed at the bottom. Let "R" represent the loop radius. Write a formula showing how the speed at the top of the loop can be calculated, assuming that the vehicle coasts to the top. (Use skill 7.)
   b. Let "S_t" represent the speed at the top, as found in part a. Show how the smallest acceptable value for that speed can be calculated from R and g. (Use skill 9.)
   c. Estimate the greatest loop diameter acceptable for this stunt. (Use skills 1 & 5.)

2. A satellite orbits at a distance of 6.0×10^7 meters (± 1%) from the center of planet X. Its period is 4.0×10^9 sec ± 2%. Calculate the satellite's centripetal acceleration. (9)

3. A cup of water at 100 degrees C is placed in a refrigerator which maintains the cup's surroundings at zero degrees. The rate at which heat energy escapes from the water is proportional to the difference in temperature between the water and its surroundings.
   a. Sketch a graph of the water's temperature vs. time. (skills 10 & 11)
   b. After two seconds the water temperature is 99.5 degrees. How long does it take for the water temperature to fall from 100 to 50 degrees? (skill 12)

4. A capacitor is initially uncharged. For 2.0 seconds its leads carry a steady 0.50-amp current. During the next two seconds the current is 0.15 A in the same direction. During the final two seconds the capacitor voltage remains at 6.40 volts.
   a. Sketch the capacitor's voltage vs time graph for the full six seconds. (skill 15b)
   b. Calculate the voltage at the end of the first section on that graph. (skills 2 & 14)
   c. Calculate the slope of the middle section on that graph. (skills 14 & 15)

5. A 6000-ohm voltmeter is connected across a previously-charged capacitor. The voltage reading decreases almost steadily from 3.8 volts to 3.5 volts in 0.15 sec.
   a. How much charge flowed through the voltmeter during that time interval? (skill 16)
   b. How much energy was released from the capacitor? (skill 17)
   c. How much energy remains stored in the capacitor at the end of that interval? (18)

6. Suppose the discharge process in #5 is allowed to continue until another batch of charge equal to the first one has passed through the voltmeter:
   a. Does this require more time, less time, or the same amount of time as the first batch?
   b. Calculate the final voltage for the second batch. (skill 14)
   c. Calculate the energy delivered by the second batch. (skill 17)

7. A 0.10-farad capacitor is connected to a generator which produces a voltage vs time graph like a cosine curve with an amplitude of 5.00 volt and a period of 0.016 second.
   a. Calculate the maximum rate at which this capacitor's voltage changes. (skill 15b)
   b. Sketch the capacitor's current vs time graph. (skills 15 & 21)
   c. Calculate the peak value of the capacitor current in this circuit. (15)
   d. Exactly how will the current amplitude be affected if we triple the generator frequency? (23)

8. A certain filter capacitor's voltage increases by 0.5% in one millisecond. Then it decreases steadily to its previous value in one sixtieth of a second while discharging. That cycle is repeated periodically. The load current is 0.16 A while it is discharging.
   a. By how much does the capacitor charge increase during each charging period? (16)
   b. Show (with a formula) how that ΔQ value can be used with the given data to estimate the maximum capacitor current. (16)
   c. The load resistance is 300 ohms. How much capacitance must this filter have? (14, 20)

ANSWER SPACE

1a) ____________
1b) ____________
1c) ____________
2) ____________
3a) ____________
3b) ____________
4a) ____________
4b) ____________
4c) ____________
5a) ____________
5b) ____________
5c) ____________
6a) ____________
6b) ____________
6c) ____________
7a) ____________
7b) ____________
7c) ____________
7d) ____________
8a) ____________
8b) ____________
8c)

SECOND APRIL TEST (1999, LEVEL H) *Solutions*

1. A new car is driven around a vertical loop on a special track so that it is upside-down at the highest point. At that moment the normal force exerted on the car by the road will be downward. That normal force cannot be less than zero, so the car's centripetal acceleration at the top must be greater than "g". The car's top speed is only 35 m/s on the level roadway approaching the loop. The stunt driver wants to be able to coast all the way around the loop.

a. Let "S_b" represent the speed at the bottom. Let "R" represent the loop radius. Show how the speed at the top of the loop can be calculated if the vehicle coasts to the top. (1 point)

Gain in altitude = 2R, so gain in PE = 2mgR = loss in KE = $(m/2)[S_b^2 - S_t^2]$
Canceling the mass and solving for the unknown speed, $\mathbf{S_t = (S_b^2 - 4gR)^{1/2}}$

b. Let "S_t" represent the speed at the top, as found in part a. Show how the smallest acceptable value for that speed can be calculated from R and g. (1 point)

The smallest acceptable centripetal acceleration is "g", so $S_t^2/R = g$, *so* $\mathbf{S_t = (Rg)^{1/2}}$

c. Estimate the greatest loop diameter acceptable for this stunt. (2 points)

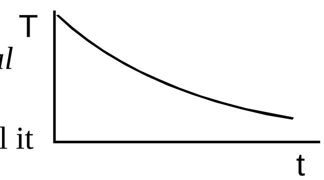
Setting the two S_t *formulas equal to each other and solving for R,* $R = S_b^2/5g$
Doubling that radius to get the diameter, $D = (2)(35\ m/s)^2/[(5)(9.8\ m/s^2)]$ = ***50 m***

2. A satellite orbits at a distance of 6.0×10^7 meters (± 1%) from the center of planet X. Its period is 4.0×10^9 sec ± 2%. Calculate the satellite's centripetal acceleration. (3 points)

$a = R(2\pi/P)^2 = (6.0 \times 10^7\ m \pm 1\%)[(2\pi)/(4.0 \times 10^9\ s \pm 2\%)]^2 =$ $\mathbf{14.8 \times 10^{-11}\ m/s^2 \pm 5\%}$

3. A cup of water at 100 deg C is placed in a refrigerator which maintains the cup's surroundings at zero degrees. The rate at which heat energy escapes from the water is proportional to the temperature difference between the water and its surroundings.

a. Sketch a graph of the water's temperature vs. time. (1 point)



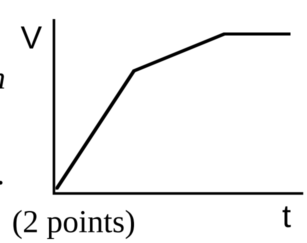
This must be an exponential process because the rate is proportional to the value as the temperature approaches equilibrium.

b. After two seconds the water temperature is 99.5 deg. How long will it take for the temperature to fall from 100 to 50 deg? (2 points)

Since a short segment of an exponential curve is fairly linear we know that it will take about 4 sec. to go down one degree, or one percent of the way to completion. We also know that it takes about 69 one-percent changes to reach the halfway point. 69 x 4 sec = ***276 seconds***

4. A capacitor is initially uncharged. For 2.0 sec its leads carry a steady 0.50-A current. During the next two seconds the current is 0.15 A in the same direction. During the final two seconds the capacitor voltage remains at 6.40 volts.

a. Sketch the capacitor's voltage vs time graph for the full six seconds. (2 points)



By the second capacitor law the slope of the voltage vs time graph is proportional to the current, which has a constant value for 2 seconds and a smaller constant value for the next 2 seconds. It is given that the voltage remains constant for the last 2 seconds.

b. Calculate the voltage at the end of the first section on that graph. (2 points)

The voltage is proportional to the charge delivered. The fraction delivered in the first two seconds is (0.5)/(0.5 + 0.15) = 0.769, so V = 0.769 x 6.40 V = ***4.92 V***

c. Calculate the slope of the middle section on that graph. (2 points)

(6.40 V - 4.92 V)/(2.0 sec) = ***0.738 V/s***

5. A 6000-ohm voltmeter is connected across a previously-charged capacitor. The voltage reading decreases almost steadily from 3.8 volts to 3.5 volts in 0.15 sec.

a. How much charge flowed through the voltmeter during that time interval? (2 points)

Average voltage = 3.65 V. Average current = Avg. Voltage/Resistance = 3.65 V/6000 ohm.
Charge delivered = avg current x time interval = 0.608 mA x 0.15 s = ***9.1 x 10^{-5} Coulomb***

b. How much energy was released from the capacitor? (2 points)

(avg V) x (charge delivered) = (3.65 J/C) x (9.1 x 10^{-5} C) = ***3.32 x 10^{-4} J***

c. How much energy remains stored in the capacitor at the end of that interval? (2 points)

If we knew the capacitance we could use $CV^2/2$ to get the stored energy. We can get the capacitance value easily because it is the slope of the charge vs voltage graph. (ΔV is given and we have already found ΔQ) So the remaining energy is:

$(\Delta Q/\Delta V)(V^2/2) = [(9.1 \times 10^{-5}\ C)/(0.3\ V)][(3.5\ V)^2/2] =$ ***1.9 x 10^{-3} J***

6. Suppose the discharge process in #5 is allowed to continue until another batch of charge equal to the first one has passed through the voltmeter:

a. Does this require more time, less time, or the same amount of time as the first batch?

The average voltage for the second batch will be less, so the current will be less, so it will take ***more time.*** (one point)

b. Calculate the final voltage for the second batch. (skill 14)

Since the new ΔQ is the same as the old one and the Q-V graph is linear, the new ΔV must be the same as the old one. 3.5 V - 0.3 V = ***3.2 V***

c. Calculate the energy delivered by the second batch. (2 points)

The new average voltage is (3.5 + 3.2)/2 = 3.35 V. The energy delivered is that voltage multiplied by ΔQ: (3.35 V)(9.1 x 10^{-5} C) = ***3.0 x 10^{-4} J***

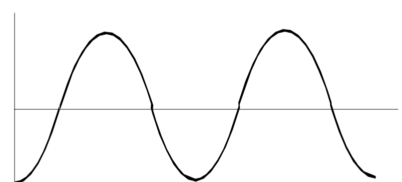
7. A 0.10 F capacitor is connected to a generator which produces a voltage vs time graph like a cosine curve with an amplitude of 5.00 V and a period of 0.016 sec.

a. Calculate the maximum rate at which this capacitor's voltage changes. (2 points)

Maximum slope = 2πA/P = (6.28) x (5.00 V) / (0.016 s) = ***1960 V/s***

b. Sketch the capacitor's current vs time graph. (1 point)

Capacitor current is proportional to the rate at which V changes. Differentiating a cosine shifts it 1/4 cycle to the left.



c. Calculate the peak value of the capacitor current in this circuit. (2)

(0.016 C/V)(1960 V/s) = ***31.4 A***

d. Exactly how will the current amplitude be affected if we triple the generator frequency? (23)

That triples the maximum slope, so the current amplitude is also ***tripled.***

8. A certain filter capacitor's voltage increases by 0.5% in one millisecond. Then it decreases steadily to its previous value in one sixtieth of a second while discharging. That cycle is repeated periodically. The load current is 0.16 A while it is discharging.

a. By how much does the capacitor charge increase during each charging period? (2 points)

Charging amount = discharging amount = 0.16 A x 0.0166 sec = ***0.00267 Coulomb***

b. Show (with a formula) how that ΔQ value can be used with the given data to estimate the maximum capacitor current. (1 point)

Maximum current = ***ΔQ/Δt****, where Δt is the one-millisecond charging time.*

c. The load resistance is 300 ohms. How much capacitance must this filter have? (14, 20)

The given load current equals the average load voltage over its resistance,
so the average load voltage = (300 V/A)(0.016 A) = 4.8 V.
The change in voltage is 0.5% of that amount: 0.005 x 4.8V = 0.024 V.
The capacitance is ΔQ/ΔV = (0.00267 C) / (0.024 V) = ***0.111 Farad***